Zbl 0534.01001

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Geometry and algebra in ancient civilizations. (English)

Berlin etc.: Springer-Verlag. XII, 223 p., 98 figs. DM 78.00; $ 32.30 (1983).

The book is meant to be the first volume of two dealing with the history of algebra, understood as mathematical procedures which can be described algebraically. At some points, but not systematically, the adequacy of the algebraic readings of the ancient texts as interpretation of the original mode of thought is also discussed. Since Ancient "algebra" is often connected to geometry, the scope of the investigation is broadened to cover aspects of geometry. The main aim of the book is, apart the descriptions of single fields of mathematical practice, to establish connections between various mathematical cultures: Babylonia, Egypt, Late Neolithic Britain (the henges), Ancient China, Ancient Greece, Ancient and Early Medieval India. No attempt is made to offer complete portraits of the single cultures or to sketch their development; the book is not meant as a replacement for the author's classical "Science Awakening" (1954; Zbl 0056.242).
Chapters 1 and 2 deal with mathematics in Late Neolithic Britain, Han- China, Egypt, Late Vedic India and Babylonia, with excurses to Greece. On the basis of Pythagorean triangles, procedures to calculate Pythagorean triples, certain second-degree problems and the ways to calculate certain areas and volumes (all shared between two or more of the cultures in question) it is argued that all these varieties of mathematics descend from a common ancestor developed by Neolithic Indo-Europeans in Central Europe - a thesis first defended by the author in Arch. Hist. Exact Sci. 23, 27-46 (1980; Zbl 0447.01002). The argument builds on the presuppositions that independent discovery and development are next to impossible not only in mathematics (p. 10) but also concerning teaching methods, technology, and social structure (e.g. p. 23); that the European Indo-Europeans and the Greeks were mathematically creative, while the other cultures were only able to translate, transmit and eventually impoverish, possessing no mathematicians but only scribes (e.g. p. 64f); and, implicitly, that the only possible relationship is common ancestry. The evidence used together with these axioms is clearly stated, but possible alternative explanations (e.g. the well-established cultural imports to Early Han-China from the Irano-Hellenico-Indian melting pot in Sogdiana) go unmentioned. So does much counterevidence, e.g. the use of "almost Pythagorean" triangles in some henges, which seems to undermine the author's belief in use of theoretical arithmetic by the henge- builders (see *J. E. Wood*, Sun, Moon and Standing Stones, Oxford 1978, pp. 39,41,51; this book is the author's main basis for his discussion of the henges).
Chapter 3 states and explains the interpretation of Elements II, Apollonios' Conics, the "application of areas" etc. as "geometric algebra", and presents the theory of proportions as a distinct algebraic theory. The presumed "geometrization of algebra" is explained as a combination of Babylonian influence with a geometricrope-stretchers' tradition brought to Greece from Egypt by Thales and Pythagoras. Chapter 4 presents the methods and problem-types of Diophantos' Arithmetics and some related works. Chapter 5 deals with linear Diophantine and Pell equations, mainly as they are solved by Āryabhaṭa I and Brahmagupta. The Indian's use of the "Euclidean algorithm" (their "pulverizer") and documented Greek solutions to single Pell-type problems are used as evidence that the Indians copied Greek originals.
Chapter 6 discusses various branches and examples of "popular mathematics" from Babylonia, Middle Kingdom and Demotic Egypt (which is shown to depend in part on Babylonian methods) and Greece. The chapter concludes by a presentation of Heron and the Hebrew Mishnat ha-Middot. The final chapter presents the mathematics of Liu Hui and Āryabhaṭa I and discusses signs of Greek influence in their work.
The book is written in the most clear and pedagogical style.

Postscript: After reading the above, Professor van der Waerden dissociated himself (with emphasis and examples) from all ideas that Babylonians, Chinese etc. should in general be unable to produce great mathematicians. I am happy to have the opportunity to precise that the arguments in chapter 1-2 on the exclusive role of non-creative scribes in Babylonian and early China are thus to be read as evaluations of particular cultures in particular periods, with no racist or other general implications.

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Keywords : algebraic procedures; Pythagorean triangles; geometric algebra; area and volume calculations; Apollonios; Pythagoras; Diophantos; Āryabhaṭa I; Brahmagupta
Classification :

- ∗01A05 General histories, source books
- 01-02 Research monographs (history)
- 01A15 Pre-Greek mathematics of Europe
- 01A25 Chinese mathematics
- 01A20 Greek or Roman mathematics
- 01A32 Indian mathematics